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$$BM: l\alpha + n\gamma = 0, \quad BQ: p\alpha + r\gamma = 0,$$

$$CN: l\alpha + m\beta = 0, \quad CR: p\alpha + q\beta = 0,$$

$$RM: lp\alpha + lq\beta + np\gamma = 0,$$

$$QN: lp\alpha + mp\beta + lr\gamma = 0.$$

At the intersections of  $BM$  and  $CN$ , of  $BQ$  and  $CR$ , and of  $RM$  and  $QN$ , the coördinate ratios are respectively:

$$\alpha_1 : \beta_1 : \gamma_1 :: -mn : nl : lm,$$

$$\alpha_2 : \beta_2 : \gamma_2 :: -qr : rp : pq,$$

$$\alpha_3 : \beta_3 : \gamma_3 :: l^2rq - p^2mn : p^2nl - l^2rp : p^2lm - l^2pq.$$

The vanishing of the determinant of these three sets of coördinates proves the proposition.

Also solved by NATHAN ALTHILLER, H. C. FEEMSTER, and F. M. MORGAN.

**444. Proposed by S. A. COREY, Hiteman, Iowa.**

Let  $ABCDE$  be a pentagon, plane or gauche, with sides  $AB$ ,  $BC$ ,  $CD$ ,  $DE$ , and  $EA$ . Bisect  $BC$  and  $DE$  in  $H$  and  $K$  respectively. Extend  $AB$  from  $B$  to  $B'$ , and  $AE$  from  $E$  to  $E'$ . On  $AB'$  take sects  $AP$  and  $AV$ , and on  $AE'$  take sects  $AL$  and  $AT$ . Draw  $AD$ ,  $AC$ ,  $AH$ ,  $AK$ , and  $DT$ . Let  $a$ ,  $b$ ,  $c$ , and  $d$  equal  $AL/AE$ ,  $AT/AE$ ,  $AV/AB$ , and  $AP/AB$ , respectively. Extend (or contract)  $AC$  from  $C$  to  $W$ , and  $AD$  from  $D$  to  $S$ , making  $AW = a \times AC$  and  $AS = d \times AD$ . Draw  $LM$  and  $PN$  parallel to, and of the same currency as,  $AD$  and  $AC$  respectively, and of lengths  $c \times AD$  and  $b \times AC$ , respectively. Draw  $AM$ ,  $AN$ ,  $ST$ , and  $WV$ . Draw  $DQ$  and  $VX$  parallel to, and of the same currency as,  $CB$  and  $TS$ , respectively. We are to prove that  $2(ad + bc)(AK \times AH \times \cos KAH + KE \times HC \times \cos QDK) = AM \times AN \times \cos MAN + TS \times VW \times \cos WVX$ .

SOLUTION BY THE PROPOSER.

Let  $KE$ ,  $HB$ ,  $AH$  and  $AK$ , in the proposed figure, be represented by the vectors  $x$ ,  $y$ ,  $z$ , and  $w$ , respectively. Then by vector addition,  $w + x = AE$ ,  $w - x = AD$ ,  $z + y = AB$ ,  $z - y = AC$ ; by construction,  $\angle ST \cdot WV = \angle WVX$ ,  $\angle KE \cdot HB = \angle QDK$ ,  $(w + x)a = AL$ ,  $(w - x)c = LM$ ,  $(z + y)d = AP$ ,  $(z - y)b = PN$ ,  $(w + x)b = AT$ ,  $(w - x)d = AS$ ,  $(z + y)z = AV$ , and  $(z - y)a = AW$ ; also by vector addition,  $(w + x)a + (w - x)c = AM$ ,  $(z + y)d + (z - y)b = AN$ ,  $(w + x)b - (w - x)d = ST$ , and  $(z + y)c - (z - y)a = WV$ .

Consider now the algebraic identity,

$$(w + x)a + (w - x)c][(z + y)d + (z - y)b] + [(w + x)b - (w - x)d][(z + y)c - (z - y)a] = 2(ad + bc)(wz + xy) \quad (1)$$

and note that when fully expanded each term is of the second degree in  $x$ ,  $y$ ,  $z$ , and  $w$ . Also note that the identity may be written

$$AM \times AN + ST \times WV = 2(ad + bc)(AK \times AH + KE \times HB) \quad (2)$$

if we substitute vectors as above indicated.

Inasmuch as vector multiplication is commutative if no term of the product is of a degree higher than the second in the vectors employed, and if the scalar part only of the resulting product be considered, we may assume that the algebraic identity (1) has a geometric interpretation which may be derived from (2) by considering the scalar part only of the vector products indicated in (2). The scalar part of the product of two vectors may be taken as the positive product of the lengths of the vectors into the cosine of their included angle. Placing this interpretation on the scalar part of the products indicated in (2) the equation of the problem is at once obtained, and the truth of the theorem established.

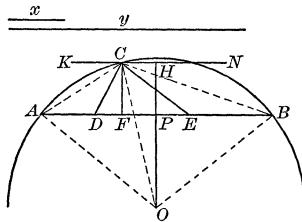
As no assumption has been made restricting any of the lines to any one plane the pentagon may, of course, be either plane or gauche. To help form a mental picture of a gauche pentagon, let  $ABCD$  be a tetrahedron, with edges  $AB$ ,  $BC$ ,  $CD$ ,  $DA$ ,  $AC$ , and  $BD$ , and let  $E$  be a point within or without such tetrahedron. Then if  $E$  be connected with  $A$  and  $D$  by right lines the figure  $ABCDE$  will be a gauche pentagon.

**445. Proposed by CLIFFORD N. MILLS, South Dakota State College.**

Given the perimeter of a right triangle and the perpendicular falling from the right angle on the hypotenuse, to determine the sides of the triangle.

**SOLUTION BY G. I. HOPKINS, Manchester (N. H.) High School.**

Let  $x$  be the altitude and  $y$  the perimeter. Draw  $AB = y$ , and  $OH$  its  $\perp$  bisector. Make  $PH = x$ , and draw  $KN$  through  $H$   $\perp$  to  $HO$ . Make  $PO = AP$ . With  $O$  as center and radius  $OA$  describe the circle  $ACB$ . Draw the chords  $CA$  and  $CB$ , and the radii  $OA$ ,  $OB$ , and  $OC$ . Make  $\angle ACD = \angle CAD$ , and  $\angle BCE = \angle CBE$ .  $\therefore DCE$  is the  $\triangle$  required. For,



$$\angle AOP = 45^\circ = \angle POB. \therefore \angle AOB = 90^\circ.$$

$\angle OAC = \angle OCA$  and  $\angle CAD = \angle ACD$ .  $\therefore \angle OCD = \angle OAD = 45^\circ$ . In like manner  $\angle OCE = 45^\circ$ ;  $\therefore \angle DCE = 90^\circ$ ,  $AD = DC$ , and  $EB = CE$ ;  $\therefore$  the perimeter of the  $\triangle DCE = y$ ,  $CF$  is  $\perp$  to  $AB$ ,  $\therefore CF = HP = x$ .

*Note.* The figure is not accurate as  $OP$  is not made equal to  $AP$ .

Also solved by B. J. BROWN, A. H. HOLMES, C. N. SCHMALL, and NATHAN ALTSCHILLER.

#### CALCULUS.

**356. Proposed by F. B. FINKEL, Drury College.**

A steel girder  $l$  ft. long and  $w$  ft. wide is moved along a passageway  $a$  ft. wide and into a corridor at right angles to the passageway. How wide must the corridor be to admit the girder?